HYPER VERSION

Received: *August 19, 1998*, Accepted: *September 9, 1998*

Strings and string breaking in 2+1 dimensional nonabelian theories

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Abstract: We consider properties of confining strings in 2+1 dimensional $SU(2)$ nonabelian gauge theory with the Higgs field in adjoint representation. The analysis is carried out in the context of effective dual Lagrangian which describes the dynamics of 't Hooft's Z_N vorices. We point out that the same Lagrangian should be interpreted as an effective Lagrangian for the lightest glueballs. It is shown how the string tension for a fundamental string arises in this description. We discuss the properties of the adjoint string and explain how its breaking occurs when the distance between the charges exceeds a critical value. The interaction between the fundamental strings is studied. It is shown that they repell each other in the weak coupling regime. We argue that in the confining regime (pure Yang-Mills theory, or a theory with a heavy Higgs field) the strings actually attract each other and the crossover between the two regimes corresponds to the crossover between the dual superconductors of first and second kind.

Keywords: Confinement, Duality in Gauge Field Theories.

JHEP09(1998)003

Contents

1. Introduction

Confinement is certainly one of the most striking qualitative features of nonabelian gauge theories. Still a reasonable understanding of this phenomenon is sorely missing. Although several possible mechanisms have been discussed over the years, we do not have a good qualitative understanding derived from a nonabelian gauge theory itself. The dual superconductivity framework [1, 2] is currently the most popular way to view the phenomenon of confinement. Lattice studies of magnetic monopoles in recent years seem to point to relevance of the monopole condensation to confining properties of nonabelian gauge theories [3]. The exact solution of $N = 2$ SUSY Yang Mills theory also indicates that the very same perturbation that leads to confinement also leads to magnetic monopole condensation [4].

There are however many unanswered questions pertaining even to the mere concept of a magnetic monopole in Yang Mills theories, let alone to the mechanism of the monopole condensation. Suffice it to say that no reasonable gauge invariant definition of a monopole is known in pure Yang Mills theories.

It therefore seems reasonable to keep an open mind on this issue and try and explore other possibilities. One such alternative mechanism of confinement was proposed long time ago by 't Hooft [5]. According to it confinement arises due to condensation of magnetic Z_N vortices. These vortices are interpolated by the 't Hooft disorder operator V which has a nontrivial commutation relations with the Wilson loop

$$VW(C) = e^{i\frac{2\pi}{N_c}n}W(C)V\,, \tag{1.1}$$

where n is the linking number between the (closed) loop C and the curve on which the disorder operator V is defined (which is a line in $3+1$ dimensions and a point in $2+1$ dimensions). It follows directly from this commutation relation that if the vacuum of a nonabelian theory is dominated by configurations with strongly fluctuating number of magnetic vortices which are only correlated on some finite distance scale, a large Wilson loop must decay according to the area law.

An apparent problem with this mechanism is that it fails to account for the properties of the adjoint Wilson loop, and in general of any Wilson loop defined in a representation with zero N-ality. Since the operator V commutes with such a Wilson loop, it does not provide for any explanation of approximate area law behavior of adjoint Wilson loops that has been observed in lattice simulations and which is required on general grounds in the large N limit[6, 7]. Recently however there has been renewed interest in this scenario due to the results of lattice Monte Carlo studies which found impressive correlations between confining properties and the properties of the Z_N vortices [8, 9]. Some considerations have also been put forward to the effect that the approximate area law behavior of the adjoint Wilson loop might be accounted for by finite thickness of the Z_N vortices [10].

Some years ago we have discussed in detail the effective dual Lagrangian description of $2+1$ nonabelian gauge theories [11]. In this approach instead of studying a gauge theory in terms of vector potentials, one considers the Lagrangian for low energy description directly in terms of the 't Hooft field V. This description has some very attractive features. It is a simple local Lagrangian for a single complex scalar field V. The detailed structure of the couplings is explicitly calculable in the weak coupling (partially broken) regime. The charged states in this picture are topological solitons of the field V. The confinement of these states is understood very simply and vividly on the classical level. It is a direct and unequivocal consequence of the fact that the dual Lagrangian is not invariant under the phase rotation of the field V, so that even though V has a nonvanishing VEV, the vacuum manifold is not continuous but consists of discrete number of points. In these circumstances the flux emanating from a charged state has to flow inside a quasi one - dimensional spatial region hence producing a flux tube [11].

Since in $2+1$ the weakly coupled (partially broken Higgs) phase is connected analytically to the strongly coupled (confinement) phase [12] one expects that the main features of the dual description are the same also in the confining region. Quantitatively however the picture in the confining phase may be somewhat different. It is precisely the purpose of this paper to extend the dual description to the confining region and to explore in this language the main qualitative differences between the weakly coupled and the strongly coupled regime. In this paper we limit ourselves for simplicity to the $SU(2)$ gauge theory with the Higgs field in adjoint representation, the so called Georgi–Glashow model. We believe however that with minimal changes the discussion should also apply to $SU(N)$ theory at any N.

The structure of this paper is the following. In section 2 we recap the construction of the dual Lagrangian and explain the significance of different terms in it. We discuss the relation of the parameters in this Lagrangian with the original couplings of the gauge theory and argue that certain parameter range corresponds to the limit in which the mass of the Higgs field becomes infinite. In this strongly coupled regime the dual Lagrangian should be viewed simply as the effective Lagrangian describing the dynamics of two lightest glueballs.

In section 3 we investigate the structure of the fundamental string. We explicitly construct in the dual theory the Wilson loop operator and show how to calculate its average. We show that the string tension is equal to the tension of the domain wall which separates two degenerate vacua which exist in this theory. This is precisely the picture advocated by 't Hooft a long time ago [5]. The adjoint string is also considered and it is shown that, up to some distance scale, the adjoint string indeed has a nonvanishing string tension. However if the string is too long, it breaks due to creation of a soliton - antisoliton pair, which in this theory has finite core energy. It should be noted that in the extreme weak coupling limit which corresponds to the compact $U(1)$ theory, the core energy of the soliton becomes infinite (of the order of the ultraviolet cutoff) and the adjoint string therefore is stable.

In section 4 we study the structure and the interaction between the fundamental strings. We find that in the weakly coupled regime the fundamental strings repel each other. The vacuum is therefore reminiscent of the superconductor of the second kind. The tension of the adjoint string is twice that of the fundamental one. In general a representation built out of n fundamental representation has a string tension approximately

$$\sigma_n = n\sigma_F \,, \tag{1.2}$$

where σ_F is the string tension of the fundamental representation. This is the same result as found in [13]. Closer to the confining regime the repulsion between the strings decreases. In fact we argue that the crossover to the confining regime happens when the interaction between the strings changes sign. As a consequence the string tension in this region should behave as

$$\sigma_n = f(n)\sigma_F, \qquad 1 < f(n) < n \,. \tag{1.3}$$

In the extreme situation when the attraction between the strings is large, the string tension should be independent of the representation, $f(n) \to 1$. It is however unlikely that the pure Yang Mills theory lies close to this point, and we would expect for pure Yang Mills theory dependence of the general type eq.(1.3).

Finally section 5 is devoted to some further discussion.

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2. The effective dual Lagrangian and its relation to the low energy excitations

We consider the $SU(2)$ gauge theory with one adjoint Higgs field — the so called Georgi–Glashow model

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2}(\mathcal{D}^{ab}_\mu H^b)^2 + \tilde{\mu}^2 H^2 - \tilde{\lambda}(H^2)^2\,, \tag{2.1}$$

where

$$\mathcal{D}^{ab}_\mu H^b = \partial_\mu H^a - e f^{abc} A^b_\mu H^c\,. \tag{2.2}$$

At large and positive $\tilde{\mu}^2$ the model is weakly coupled and perturbative description is valid. The $SU(2)$ gauge symmetry is broken down to $U(1)$ and the Higgs mechanism takes place. Two gauge bosons, $W^\pm$, acquire a mass, while the third one, the "photon", remains massless to all orders in perturbation theory. It is well known [14] that beyond the perturbation theory the photon also acquires an exponentially small mass $m^2_{ph} \propto M^2_W \exp\{-M_W/e^2\}$. The same mechanism leads to confinement of the charged gauge bosons with a tiny nonperturbative string tension $\sigma \propto e^2 m_{ph}$.

For negative $\tilde{\mu}^2$ the gauge symmetry is unbroken, the theory is strongly interacting and the standard confinement phenomenon is therefore expected. At large negative $\tilde{\mu}^2$ the Higgs field becomes heavy and decouples, and the theory reduces to pure Yang Mills theory. Although the spectrum in the strong coupling regime and in the weak coupling regime are different, there is no phase transition between the two regimes but rather a smooth crossover corresponding to the fact that there is no gauge invariant order parameter which could distinguish between these two phases [12].

We will now recap how the effective dual Lagrangian is constructed in the weakly coupled phase. First, it is convenient to define a gauge invariant electric current

$$J^\mu = \epsilon^{\mu\nu\lambda}\partial_\nu \tilde{f}_\mu, \qquad Q = \int d^2x\, J_0(x)\,. \tag{2.3}$$

Here

$$\tilde{f}_\mu = \epsilon_{\mu\nu\lambda} F^a_{\nu\lambda}\hat{H}^a\,, \tag{2.4}$$

where $\hat{H}^a \equiv \frac{H^a}{|H|}$. Consider now the following operator:

$$V(x) = \exp\frac{i}{e}\int d^2y\, \left[\epsilon_{ij}\frac{(x-y)_j}{(x-y)^2}\hat{H}^a(y)E^a_i(y) + \Theta(x-y)J_0(y)\right]. \tag{2.5}$$

It is the operator of a singular $SU(2)$ gauge transformation with the field dependent gauge function

$$\lambda^a(y) = \frac{1}{e}\Theta(x-y)\hat{H}^a(\vec{y})\,. \tag{2.6}$$

This field dependence of the gauge function ensures the gauge invariance of the operator V. This is the explicit gauge invariant form of 't Hooft's "disorder parameter" [5]. As discussed in detail in [11], [15] the operator V is a local scalar field.

It is shown in [11] that the low energy physics of the weakly coupled phase is conveniently described in terms of the effective Lagrangian of the field V. The general structure of this Lagrangian is determined by the relevant symmetries very much in the same way as the structure of chiral Lagrangian in QCD is determined by the (spontaneously or explicitly broken) chiral symmetry. Classically the following current is conserved in this model

$$\tilde{F}^\mu = \tilde{f}^\mu - \frac{1}{e}\epsilon^{\mu\nu\lambda}\epsilon^{abc}\hat{H}_a(\mathcal{D}_\nu\hat{H})^b(\mathcal{D}_\lambda\hat{H})^c . \tag{2.7}$$

The vortex operator V is a local eigenoperator of the abelian magnetic field $B(x) = \tilde{F}_0$.

$$[V(x), B(y)] = -\frac{2\pi}{e}V(x)\delta^2(x-y) . \tag{2.8}$$

That is to say, when acting on a state it creates a pointlike magnetic vortex which carries a quantized unit of magnetic flux. The transformation generated by the magnetic flux $\Phi \equiv \int d^2x B(x)$ therefore acts on the vortex field V as a phase rotation

$$e^{i\alpha\Phi}V(x)e^{-i\alpha\Phi} = e^{i\frac{2\pi\alpha}{e}}V(x) . \tag{2.9}$$

The classical conservation of Φ is spoiled quantum mechanically by the presence of 't Hooft-Polyakov monopoles. However the discreet Z_2 subgroup of the transformation group eq.(2.9) $V \to -V$ remains unbroken in the quantum theory as well [11]. The effective Lagrangian preserves this Z_2 symmetry. Its form is

$$\mathcal{L} = \partial_\mu V^*\partial^\mu V - \lambda(V^*V-\mu^2)^2 - \frac{m^2}{4}(V^2+V^{*2}) + \zeta(\epsilon_{\mu\nu\lambda}\partial_\nu V^*\partial_\lambda V)^2 . \tag{2.10}$$

The coupling constants in eq.(2.10) are determined in the weakly coupled region from perturbation theory and dilute monopole gas approximation. In the weakly coupled region we have

$$\begin{aligned} \mu^2 &= \frac{e^2}{8\pi^2}, & m &= m_{ph}, \\ \lambda &= \frac{2\pi^2 M_H^2}{e^2}, & \zeta &\propto \frac{M_W}{e^4 M_H^2} . \end{aligned} \tag{2.11}$$

Here M_H is the Higgs mass in the original theory, M_W is the mass of the W-boson and m_{ph} is the exponentially small nonperturbative photon mass calculated by Polyakov [14].

The first term in this Lagrangian is the standard kinetic term for the field V. The second term is a potential invariant under the $U(1)$ phase rotation. As discussed above, this rotation is generated by the magnetic flux. The third term is the manifestation of the quantum mechanical breaking of the flux symmetry and its coefficient is proportional to the monopole density, hence large suppression in the weak coupling limit. Finally the last term contains four derivatives. It is the analog of the Skyrme term in the effective chiral Lagrangian of QCD. As befits a higher derivative term it is not relevant if we wish to discuss physics only at scales below the mass of $W^\pm$. We chose to keep it since we want discuss the issue of the string breaking which as we shall see is sensitive to this term.

Let us discuss some simple properties of the dual description. First, we are only going to consider the model for $\mu^2 > 0$, and therefore vacuum expectation value of V is nonvanishing. In fact according to eq. (2.11) this VEV is proportional to the gauge coupling constant. To understand why this is so let us recall how a charged state is represented in this dual description. As shown in [11] the electric current eq. (2.3) can be expressed in terms of the vortex operator eq. (2.5) in the form

$$\frac{e}{\pi} J_\mu = i\epsilon_{\mu\nu\lambda}\partial_\nu (V^* \partial_\lambda V)\,.$$

The electric charge is therefore proportional to the winding number of the phase of the field V. A charge state is a soliton of V with a nonzero winding number. Neglecting for a moment the small $U(1)$ noninvariant term in eq. (2.10), we find that the minimal energy configuration in one soliton sector is a rotationally invariant hedgehog, Fig.1, which far from the soliton core has the form

$$V(x) = \mu e^{i\theta(x)}. \tag{2.12}$$

Here $\theta(x)$ is an angle between the vector x and one of the axes. The self energy of this configuration is logarithmically divergent in the infrared due to the contribution from the kinetic term

$$E = 2\pi\mu^2 \ln M_H L\,. \tag{2.13}$$

This self energy should be equal to the electromagnetic energy of a charged state in the original description. The Coulomb part of the self energy of a charged state indeed diverges logarithmically. Matching the coefficients of the logarithmic divergence gives the first of the equations in eq. (2.11).

In this argument we have neglected the $U(1)$ noninvariant term in the Lagrangian, since its coefficient is nonperturbatively small. Without this term the Lagrangian eq. (2.10) contains a massless excitation — the phase of the field V. This is of course the photon which is indeed massless in perturbation theory.[1] It is massless since the theory has infinite number of vacua, corresponding to an arbitrary phase of the VEV of V. Due to this the energy of a charged soliton is only logarithmically divergent and there is no linear confinement.

If we now reinstate the symmetry breaking term the situation changes qualitatively. First of all the degeneracy between the different phases of V is lifted and the vacuum manifold now consists of only two points $V = \pm\mu$. Consequently, the theory is not a gapless one anymore. The lightest excitation of the Lagrangian eq. (2.10) is still the phase of V which now has a small mass m. This field also becomes self interacting with

[1] Note that in 2+1 dimensions a massless photon is a scalar particle.

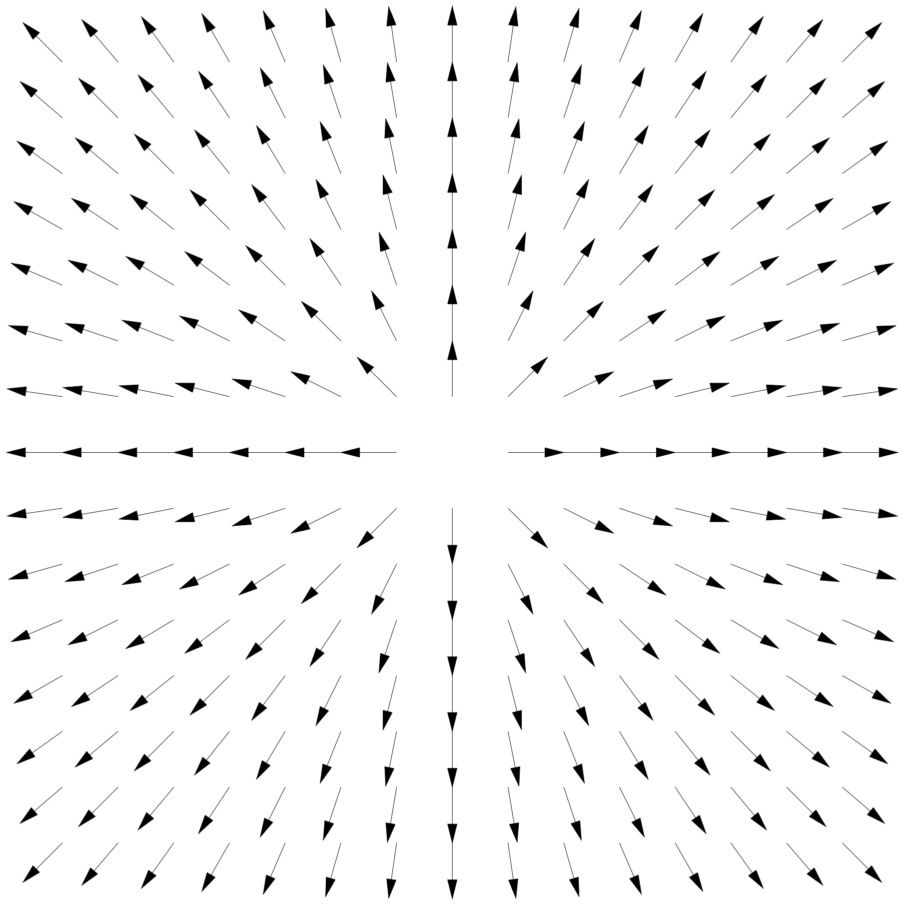

Figure 1: The hedgehog configuration of the field V in the state of unit charge when the symmetry breaking terms in the effective Lagrangian are neglected.

the sine-Gordon type potential which follows directly from the $U(1)$ noninvariant term in eq. (2.10). This is precisely the self–interaction of the massive photon calculated first in [14].

The explicit symmetry breaking causes a more dramatic change in the topologically charged (soliton) sector. The energy of an isolated soliton now diverges in the infrared *linearly* rather than just logarithmically. The configuration that forms around the soliton is a string, fig. 2, rather than a hedgehog of fig. 1. The hedgehog configuration is no longer energetically favoured because in such a configuration the phase of the field V is far from its vacuum value almost everywhere in space. The energy of a hedgehog therefore becomes quadratically divergent: $E \propto e^2m^2L^2$, where L is an infrared cutoff. To minimize the energy for a nonzero winding, the system chooses a stringlike configuration fig. 2. The phase of $V(x)$ deviates from 0 (or π) only inside a strip of width $d \sim 1/m$ stretching from the location of the defect to infinity. The energy of such a configuration is linearly divergent. This is the simple picture of confinement in the dual formulation in the weakly coupled regime.

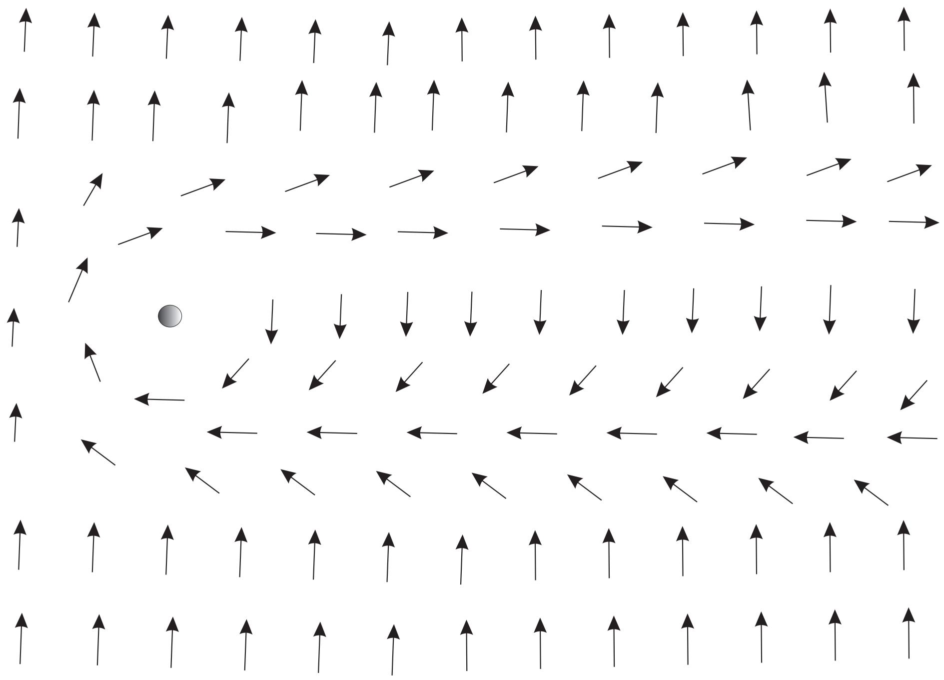

Figure 2: The stringlike configuration of the field V in the state of unit charge in the presence of the symmetry breaking terms in the effective Lagrangian.

We shall see in the next section that this discussion is appropriate for the adjoint string. The fundamental string appears in a somewhat different fashion.

Before discussing the confining properties of the theory in more detail let us make the following comment concerning the relation between fields entering the effective Lagrangian and physical excitations of the nonabelian theory.

The vortex operator as defined in eq. (2.5) has a fixed length whereas the field V which enters the Lagrangian eq. (2.10) is a conventional complex field. How should one understand that? First of all at weak gauge coupling the quartic coupling in the dual Lagrangian is large $\lambda \to \infty$. This condition freezes the radius of V dynamically. In fact even at finite value of λ if one is interested in the low energy physics, the radial component is irrelevant as long as it is much heavier than the phase. Indeed at weak gauge coupling the phase of V which interpolates the massive photon is much lighter than all the other excitations in the theory. Effectively therefore at low energies eq. (2.10) reduces to a nonlinear sigma model and one can identify the field V entering eq. (2.10) directly with the vortex operator of eq. (2.5). However it is well known that quantum mechanically the radial degree of freedom of a sigma model field is always resurrected. The spectrum of such a theory always contains a scalar particle which can be combined with the phase into a variable length complex field. The question is only quantitative — how heavy is this scalar field relative to the phase.

Another way of expressing this is the following. The fixed length field V is defined at the scale of the UV cutoff in the original theory. To arrive at the low energy effective

Lagrangian one has to integrate over all quantum fluctuations down to some much lower energy scale. In the process of this integrating out the field is "renormalized" and it acquires a dynamical radial part. The mass of this radial part then is just equal to the mass of the lowest particle with the same quantum numbers in the original theory. This is in fact why the parameters in eq. (2.11) are such that the mass of the radial part of V is equal to the mass of the scalar Higgs particle.

This brings us to the following observation. We know how to calculate the couplings of the dual Lagrangian only in the weak gauge coupling limit. However we also know that the weak and the strong coupling regimes in this model are not separated by a phase transition. It is therefore plausible that the low energy dynamics at strong coupling is described by the same effective Lagrangian. Clearly if this is the case the degrees of freedom that enter this Lagrangian must interpolate real low energy physical states of the strong coupling regime, that is to say must correspond to lightest glueballs of pure $SU(2)$ Yang Mills theory. It turns out that in fact the quantum numbers of two lightest glueballs are precisely the same as those of the phase and the radius of the vortex field V. The radial part of V is obviously a scalar and has quantum numbers 0^{++}. The quantum numbers of the phase are easily determined from the definition eq. (2.5). Those are 0^{--}. The spectrum of pure $SU(N)$ Yang Mills theory in 2+1 dimensions was extensively studied recently on the lattice [16]. The two lightest glueballs for any N are found to have exactly those quantum numbers. The lightest excitation is the scalar while the next one is a charge conjugation odd pseudoscalar with the ratio of the masses roughly $m_p/m_s = 1.5$ for any N.[2]

We are therefore lead to the following conjecture. The low energy physics of the $SU(2)$ gauge theory is always described by the effective Lagrangian eq. (2.10). In the weak coupling regime the parameters are given in eq. (2.11). Here the pseudoscalar particle is the lightest and the scalar is the first excitation. The pseudoscalar as an almost massless photon and the scalar is the massive Higgs particle. Moving towards the strong coupling regime (decreasing the Higgs VEV in the original language) corresponds to increasing the pseudoscalar mass while reducing the scalar mass and the parameters of the effective Lagrangian change accordingly. The crossover between the weak and the strong coupling regimes occurs roughly where the scalar and the pseudoscalar become degenerate. At strong coupling the degrees of freedom in the effective Lagrangian are the two lightest glueballs. They are however still collected in one complex field which represents nontrivially the exact Z_2 symmetry of the theory. We stress that the existence of the Z_2 symmetry in this model is an exact statement which is not related to the weak coupling limit. It is therefore completely natural to expect that this symmetry

[2]Actually this state of matters is firmly established only for $N > 2$. At $N = 2$ the mass of the pseudoscalar has not been calculated in [16]. The reason is that it is not clear how to construct a charge conjugation odd operator in a pure gauge $SU(2)$ lattice theory. So it is possible that the situation at $N = 2$ is nongeneric in this respect. In this case we would hope that our strong coupling picture applies at $N > 2$.

must be nontrivially represented in the effective low energy Lagrangian. Of course the spectrum of pure Yang Mills theory apart from a scalar and a pseudoscalar glueballs contains many other massive glueball states and those are not separated by a large gap from the two lowest ones. Application of this effective Lagrangian to the strong coupling regime therefore has to be taken in a qualitative sense.

In the rest of this paper we will explore the consequences of this picture on the properties of confinement. We will see that although in both regimes the confining strings arise naturally in the effective Lagrangian description, their properties depend in an important way on whether the lightest particle is a scalar or a pseudoscalar.

3. String tension and the breaking of the adjoint string

We now turn to the study of the confining properties of the theory. In the preceding section we have briefly described why the charged states have linearly IR divergent energy. In this section the discussion will be made slightly more formal. First we want to discuss the fundamental string and its string tension. For this we need to know how to calculate the expectation value of a fundamental Wilson loop in the dual theory. We therefore start by constructing the Wilson loop operator.

Let us first consider a spacelike Wilson loop. The vortex operator eq. (2.5) and the fundamental Wilson loop operator satisfy the 't Hooft commutation relation

$$W(C)V(x) = V(x)W(C)e^{i\pi n(C,x)}\,, \tag{3.1}$$

where $n(C,x)$ is the linking number between the loop C and the point x: $n(C,x) = 1$ if x is inside the surface bounded by C and vanishes otherwise. To see this use the fact that under a gauge transformation the Wilson loop transforms as

$$U^\dagger W(C)U = W(C)Pe^{\int_C dl_i U^\dagger \partial_i U}\,. \tag{3.2}$$

For regular gauge functions $U(x)$ the phase factor on the RHS vanishes and the Wilson loop is invariant. The operator V however is the operator of a singular gauge transformation. The gauge function λ^a eq. (2.6) has a discontinuity equal to π along the cut of the angular function $\theta(x)$. For this function the phase factor in eq. (3.2) is unity if the cut crosses the contour C an even number of times and equals -1 when the cut crosses C an odd number of times. This leads to the commutator eq. (3.1).

Now it is straightforward to write down an operator in terms of the field V that has the same property

$$W(C) = e^{i\pi \int_S d^2x P(x)}\,. \tag{3.3}$$

Here S is the surface bounded by the contour C, and P is the operator of momentum conjugate to the phase of V. Introducing the radius and the phase of V by

$$V(x) = \rho(x)e^{i\chi(x)}\,, \tag{3.4}$$

one can write the path integral representation for calculating the vacuum average of the Wilson loop as

$$\langle W(C)\rangle = \int DV \exp\left\{ i\int d^3x \rho^2(\partial_\mu\chi - j^S_\mu)^2 + (\partial_\mu\rho)^2 - U(V)\right\}\,, \tag{3.5}$$

where $U(V)$ is the Z_2 invariant potential of eq. (2.10). The external current $j^S_\mu(x)$ does not vanish only at points x which belong to the surface S and is proportional to the unit normal n_μ to the surface S. Its magnitude is such that when integrated in the direction of n it is equal to π. These properties are conveniently encoded in the following expression:

$$\int_T dx_\mu j^S_\mu(x) = \pi n(T,C)\,. \tag{3.6}$$

Here T is an arbitrary closed contour, and $n(T,C)$ is the linking number between two closed curves T and C. In eq. (3.5) we have neglected the four derivative term present in eq. (2.10) for simplicity. Have we kept it, the derivative of the phase field $\partial_\mu\chi$ would have been shifted by the same current j^S_μ in this term also. The expression eq. (3.5) follows directly from the operatorial definition of the Wilson loop eq. (3.3), if one properly takes care of the seagull terms. Those are responsible for the appearance of the j^2_μ term in eq. (3.5). This issue for operators similar to (3.3) is discussed in detail in [15] and [17].

So far we have considered spatial Wilson loops. However the expression eq. (3.5) is completely covariant, and in this form is valid for timelike Wilson loops as well. It is important to note that although the expression for the current depends on the surface S, the Wilson loop operator in fact depends only on the contour C that bounds this surface. A simple way to see this is to observe that a change of variables $\chi \to \chi + \pi$ in the volume bounded by $S + S'$ leads to the change $j^S_\mu \to j^{S'}_\mu$ in eq. (3.5). The potential is not affected by this change since it is globally Z_2 invariant. Therefore the operators defined with S and S' are completely equivalent.

To calculate the energy of a pair of static fundamental charges at points A and B we have to consider a timelike fundamental Wilson loop of infinite time dimension. This corresponds to time independent j_μ which does not vanish only along a spatial curve G connecting the two points and pointing in the direction normal to this curve (fig. 3). The shape of the curve itself does not matter, since changing the curve without changing its endpoints is equivalent to changing the surface S in eq. (3.5). In the classical approximation the path integral eq. (3.5) is dominated by a static configuration of V. To determine it we have to minimize the energy on static configurations in the presence of the external current j_μ. The qualitative

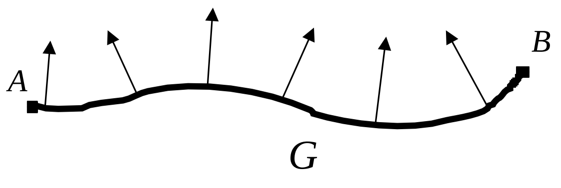

Figure 3: The external current j_μ which creates the pair of static fundamental charges in the effective Lagrangian description.

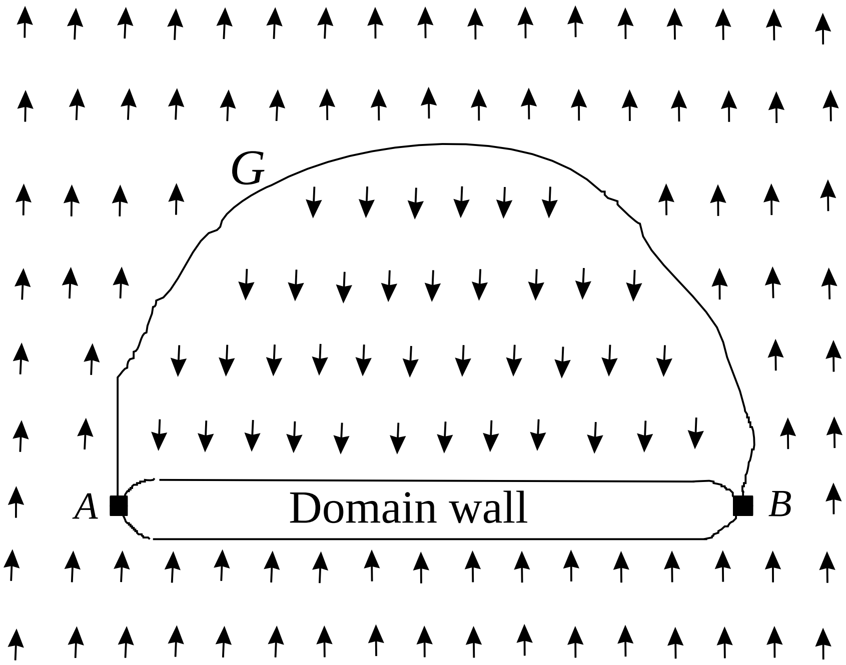

Figure 4: The minimal energy configuration of V in the presence of a pair of fundamental charges.

features of the minimal energy solution are quite clear. The effect of the external current clearly is to flip the phase of V by π across the curve G, as is expressed in eq. (3.6). Any configuration that does not have this behavior will have the energy proportional to the length of G and to the UV cutoff scale. Recall that the vacuum in our theory is doubly degenerate. The sign change of V transforms one vacuum configuration into the other one. The presence of j_μ therefore requires that on opposite sides of the curve G, immediately adjacent to G there should be different vacuum states. It is clear however that far away from G in either direction the field should approach the same vacuum state, otherwise the energy of a configuration diverges linearly in the infrared. The phase of V therefore has to make half a wind somewhere in space to return to the same vacuum state far below G as the one that exists far above G. If the distance between A and B is much larger than the mass of the lightest particle in the theory, this is achieved by having a segment of a domain wall between the two vacua connecting the points A and B. Clearly to minimize the energy the domain wall must connect A and B along a straight line. The energy of such a domain wall is proportional to its length, and therefore the Wilson loop has an area law behavior. The minimal energy solution is schematically depicted on fig. 4.

We see that the string tension for the fundamental string is equal to the tension of the domain wall which separates the two vacua in the theory. This conclusion is equally valid in the weakly and strongly coupled regimes. The structure of the domain wall and the relation between the tension and the parameters of the theory is quite different in the two regimes and we will come back to this question in the next section.

Note that the fundamental string is an absolutely stable topological object in the Z_2 invariant theory: the domain wall. It can not break, if one makes the distance between the two charges larger. From the point of view of the original theory this is so because the theory does not contain particles with fundamental charge. In the dual description it is also obvious since there is no object in the theory on which a domain wall can terminate. One way of thinking about it is in terms of the "electric charge" eq. (2.3). As discussed in the previous section this charge counts the number of windings of the field V. Across the domain wall the phase of V changes by π, therefore the winding number of a point at which the domain wall can end must be half integer. Finite energy states with half integer winding do not exist in the theory and therefore the fundamental string can not break. The only reason why the wall can end on a fundamental external charge is because of the external current j_μ which itself furnishes an extra half a wind, so that the total winding number around points A and B is still integer.

The situation is rather different if we consider adjoint external charges instead. Here we have to be more precise what we mean by that. A fundamental Wilson loop gives an energy of a state with two heavy external charges in a fundamental representation. We would now like to introduce two external charges in adjoint representation. The adjoint Wilson loop however has a trivial commutation relation with the vortex operator, and this therefore does not tell us what operator we should consider in the dual theory in order to "measure" the interaction energy of the sources. Moreover in the weakly coupled phase, where the gauge symmetry is "broken" not all components of the external adjoint charge are equivalent. The three states in the adjoint representation of $SU(2)$ split into three representation with respect to the unbroken electric charge generator eq. (2.3), with eigenvalues $0, 1$ and -1.[3] The interactions of these three states are obviously different. The neutral state does not couple directly to the photon. It is therefore not expected to feel any confining force at all and is not interesting from our point of view. The remaining two states carry electric charge and interact just like $W^\pm$ through the photon exchange. We therefore would like to concentrate on these states. In the following whenever we speak about the adjoint string we mean the string between the external charges 1 and -1.

The introduction of an external charge $Q = 1$ at a point A from the point of view of the dual theory means that we force the theory into the topological sector with unit winding of V around A. The dipole configuration corresponds to a unit wind at A and a minus one wind at B.

What is the minimum energy configuration in this sector? There are two types of configurations which should be considered (we are assuming that the distance between the external charges is much larger than the inverse mass of the lowest excitation). First there is a stringlike configuration, where the points A and B are connected by a

[3]Note that for the fundamental external charge $Q = \pm 1/2$.

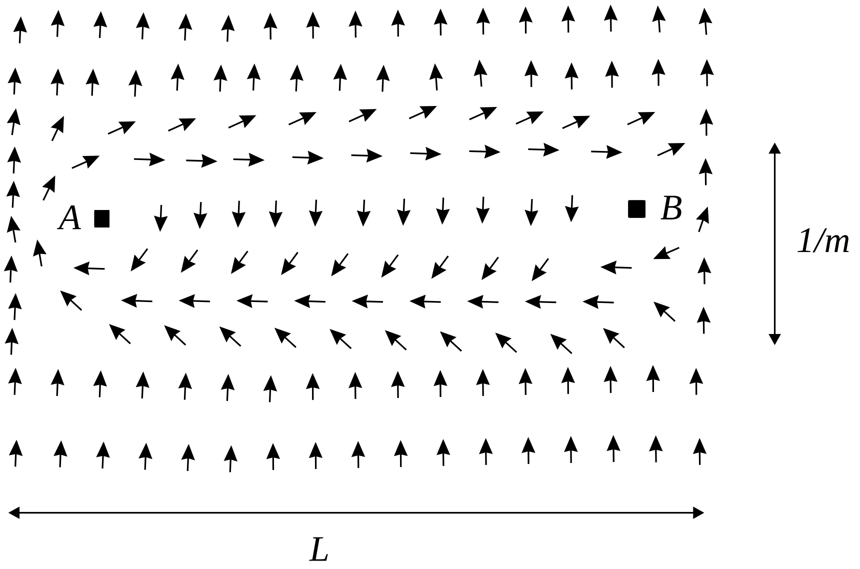

Figure 5: The stringlike configuration of V in the presence of a pair of adjoint charges. This configuration has the minimal energy if $L < L_c$.

"double domain wall", inside which the phase of the field V completes a rotation by 2π. Thus here the flux emanating from A propagates all the way to B and terminates there. This is depicted schematically on fig. 5. These are two charges of fig. 2 joined by a string.

The energy of this configuration depends linearly on the distance L. The energy density per unit length is obviously of the same order as that for the domain wall, although somewhat higher. The energy of this type of configuration is therefore

$$E_1 = x\sigma_F L\,. \tag{3.7}$$

Here x is a number of order one, and σ_F — the fundamental string tension.

Another type of configuration is where the flux emanating from A is screened locally by creating a topological soliton nearby. This is possible since in order to screen the flux the soliton has to carry an integer winding number rather than halfinteger as in the case of the fundamental charge. There are such solitonic solutions of classical equations of motion of eq. (2.10). The structure of these solutions is not difficult to understand. The radial field ρ must vanish in the core of such a configuration. The size of the core, that is the area in which ρ significantly differs from its value in the vacuum must be of order $S = 1/M_H^2$. The charge density within this area is of order $J_0 \sim \mu^2 S = e^2 M_H^2$. The energy associated with the core comes mainly from the four derivative term in eq. (2.10) and is

$$E_c \sim \zeta e^4 M_H^2\,. \tag{3.8}$$

The energy of the screened dipole configuration is given by (neglecting the Coulomb energy of the two dipoles)

$$E_2 = 2E_c\,. \tag{3.9}$$

From this analysis it is clear that as long as the distance between the external charges does not exceed the critical value $L_c \sim 2\zeta e^4 M_H^2/\sigma_F$ the energetically favourable option is a string. However at larger distances the string breaks and it is the screened dipole configuration that has lower energy.

The solitonic solution of the dual Lagrangian obviously has to be identified with the lightest charged particle in the Nonabelian gauge theory. In the weakly coupled regime this is just $W^{\pm}$. Indeed with the coupling ζ given by eq. (2.11) the core energy of the soliton is equal to the mass of the $W^{\pm}$.

At weak coupling the mass of the charged boson is related to the VEV of the Higgs field by

$$M_W \sim e\langle H\rangle\,. \tag{3.10}$$

On the other hand the string tension is determined by the mass of the photon [14]

$$\sigma_F \sim e^2 m\,. \tag{3.11}$$

We therefore have

$$L_c \sim \frac{\langle H\rangle}{e}\frac{1}{m} \gg \frac{1}{m}\,. \tag{3.12}$$

In this regime therefore the critical length is much larger than the inverse photon mass, which determines the thickness of the string. Nevertheless if the string is too long it breaks. It has been emphasized recently in [13] that the breaking of the string is a qualitative phenomenon which distinguishes the Georgi–Glashow model from the compact $U(1)$ theory. In the latter all strings are stable. In the dual Lagrangian approach this is indeed seen very clearly. The compact $U(1)$ theory limit is obtained from eq. (2.10) at $\lambda \to \infty$, $\zeta \to \infty$. In this limit the core energy of the soliton diverges and the critical length becomes infinite.

In the weak coupling regime one can reasonably speak about a well developed string long before it breaks. In the strong coupling regime the situation is less clear. We will briefly come back to this point in the next section.

4. Strong versus weak coupling and the string interaction

In this section we discuss the structure of the fundamental string and then the interaction between the strings. Consider first the weakly coupled regime. As discussed above in terms of the coupling constants in the effective Lagrangian this corresponds to the limit where the phase of V is much lighter than the radial part. A cartoon of the fundamental string in this situation is depicted in fig. 6. The radial part ρ being very heavy practically does not change inside the string. In fact the value of ρ in the middle of the string can be estimated from the following simple argument. The width of region where ρ varies from its vacuum value μ to the value ρ_0 in the middle is of the

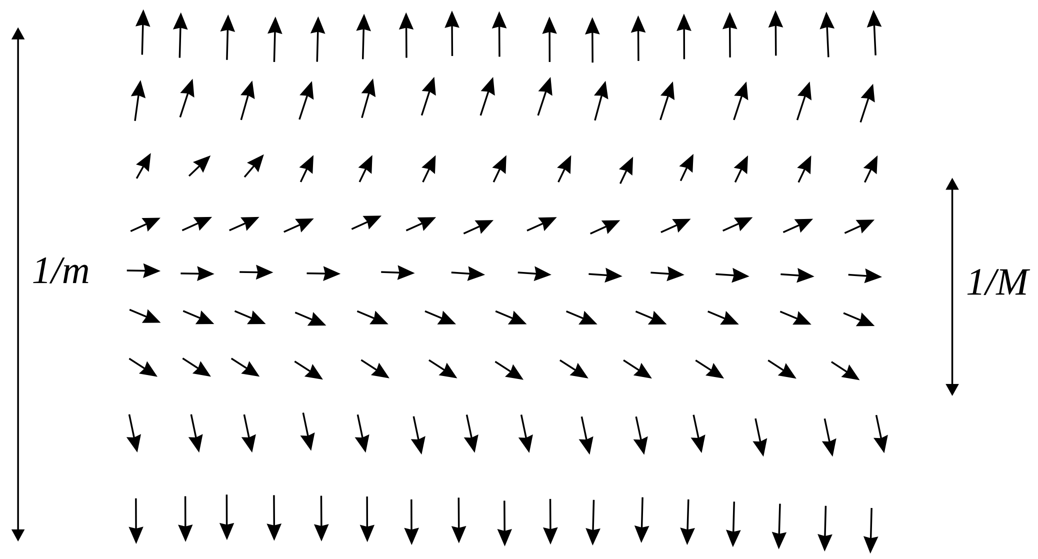

Figure 6: The structure of the string (domain wall) in the regime when the pseudoscalar is lighter than the scalar, $m < M$.

order of the inverse mass of ρ. The energy per unit length that this variation costs is

$$\sigma_\rho \sim M(\mu - \rho_0)^2 + \frac{m^2}{M}\rho_0^2\,, \tag{4.1}$$

where the first term is the contribution of the kinetic term and the second contribution comes from the interaction term between ρ and χ due to the fact that the value of χ in the middle of the string differs from its vacuum value. Minimizing this with respect to ρ_0 we find

$$\rho_0 = \mu(1 - x\frac{m^2}{M^2})\,. \tag{4.2}$$

We see therefore that even in the middle of the string the difference in the value of ρ and its VEV is second order in the small ratio m/M. Correspondingly the contribution of the energy density of ρ to the total energy density is also very small.

$$\sigma_\rho \sim \frac{m}{M}m\mu^2\,. \tag{4.3}$$

This is to be compared with the total tension of the string which is contributed mainly by the pseudoscalar phase χ

$$\sigma_\chi \sim m\mu^2\,. \tag{4.4}$$

This again we obtain by estimating the kinetic energy of χ on a configuration of width $1/m$ where χ changes by an amount of order 1.

Remembering that $\mu^2 \sim e^2$ we see that the fundamental string tension parametrically is

$$\sigma_F \sim me^2\,. \tag{4.5}$$

This is consistent with Polyakov's calculation in dilute monopole gas approximation [14].

It is worth stressing the following important feature of this simple analysis. The heavy radial field ρ practically does not contribute to the string tension. This is natural of course from the point of view of decoupling. In the limit of infinite mass ρ should decouple from the theory without changing its physical properties. It is however very different from the situation in superconductors. In a superconductor of the second kind, where the order parameter field is much heavier than the photon (correlation length is smaller than the penetration depth $\kappa > \frac{1}{\sqrt{2}}$) the magnetic field and the order parameter give contributions of the same order to the energy density of the Abrikosov vortex (up to logarithmic corrections $O(\log \kappa)$). This is the consequence of the fact that the order parameter itself is forced to vanish in the core of the Abrikosov vortex, and therefore even though it is heavy, its variation inside the vortex is large. An even more spectacular situation arises if we consider a domain wall between two vacuum states in which the heavy field has different values [18]. In this situation the contribution of the heavy field ϕ to the tension would be

$$\sigma_{heavy} = M(\Delta\phi)^2 \,, \tag{4.6}$$

where $\Delta\phi$ is the difference in the values of ϕ on both sides of the wall. For fixed $\Delta\phi$ the energy density diverges when ϕ becomes heavy. In our case this does not happen since the two vacua which are separated by the domain wall differ only in VEV of the light field χ and not the heavy field ρ.

Let us now consider the fundamental string in the opposite regime, that is when the mass of the scalar is much smaller than the mass of the pseudoscalar. The profile of the fields in the wall now is very different. The cartoon of this situation is given on fig. 7. We will use the same notations, denoting the mass of the pseudoscalar by m and the mass of the scalar by M, but now $m \gg M$. Let us again estimate the string tension and the contributions of the scalar and a pseudoscalar to it. The width of the region in which the variation of ρ takes place is of the order of its inverse mass. The estimate of the energy density of the ρ field is given by the contribution of the kinetic term

$$\sigma_\rho \sim M(\mu - \rho_0)^2 \,. \tag{4.7}$$

The width of the region in which the phase χ varies is $\sim 1/m$. In this narrow strip the radial field ρ is practically constant and is equal to ρ_0. The kinetic energy of χ therefore contributes

$$\sigma_\chi \sim m\rho_0^2 \,. \tag{4.8}$$

Minimizing the sum of the two contributions with respect to ρ_0 we find

$$\rho_0 \sim \frac{M}{m}\mu \ll \mu \,. \tag{4.9}$$

And also

$$\begin{aligned} \sigma_\chi &\sim \frac{M}{m} M\mu^2 \,, \\ \sigma_F = \sigma_\rho &\sim M\mu^2 \,. \end{aligned} \tag{4.10}$$

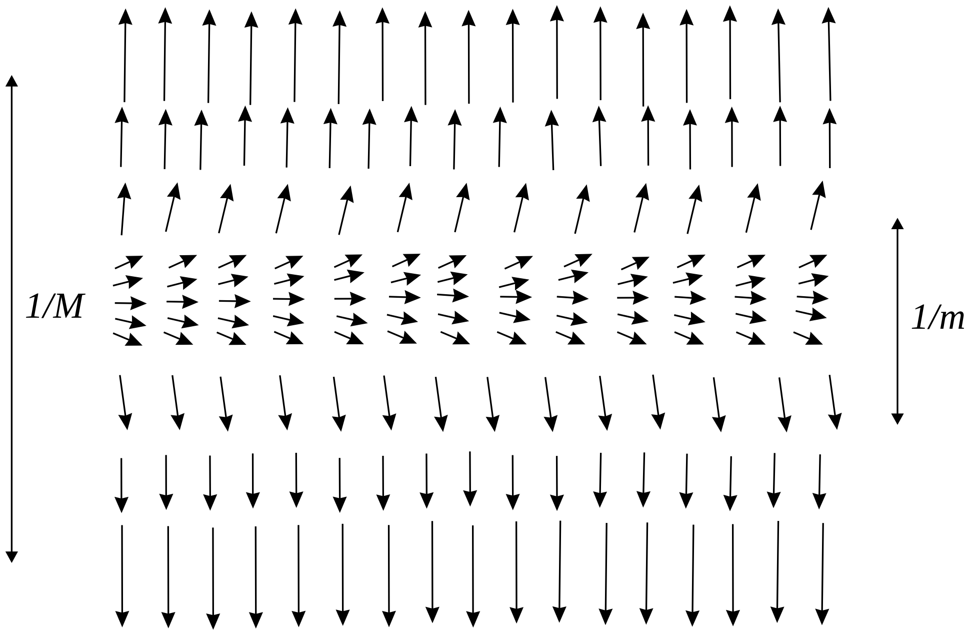

Figure 7: The structure of the string (domain wall) in the regime when the pseudoscalar is heavier than the scalar, $m > M$.

Now the radial field almost vanishes in the core of the string. The energy density is contributed entirely by the scalar rather than by the pseudoscalar field. Again this is in agreement with decoupling. The heavier field does not contribute to the energy, even though its values on the opposite sides of the wall differ by $O(1)$. Its contribution to the energy is suppressed by the factor ρ_0^2 which is very small inside the wall.

We have discussed here the extreme situation $m \gg M$. This regime is not realized in the non Abelian gauge theory. From the lattice simulations we know that in reality even in the pure Yang Mills case the ratio between the pseudoscalar and scalar masses is about 1.5 — not a very large number. The analysis of the previous paragraph therefore does not reflect the situation in the strongly coupled regime of the theory. Rather we expect that the actual profile of the string is somewhere in between fig. 6 and fig. 7 although somewhat closer to fig. 7. The widths of the string in terms of the scalar and pseudoscalar fields are of the same order, although the scalar component is somewhat wider. The same goes for the contribution to the string tension. Both glueballs contribute, with the scalar contribution being somewhat larger. Of course the spectrum of the Yang–Mills theory apart from the scalar and the pseudoscalar contains many other glueballs. Those are not included in our effective Lagrangian but their masses are in fact not that much higher than the masses of the two lowest states. They therefore also give a nonnegligible contribution to the string tension, but we have nothing to say about this in the present framework.

Having understood the structure of the domain wall in the two extreme regimes, we have to ask ourselves what is the interaction between two such domain walls, or equivalently between two fundamental confining strings. The answer to this question is straightforward. In the weakly coupled region we can disregard the variation of ρ. For two widely separated strings the interaction energy comes from the kinetic term of χ. This obviously leads to repulsion, since for both strings in the interaction region the derivative of the phase is positive. On the other hand if the pseudoscalar is very heavy the main interaction at large separation is through the "exchange" of the scalar. This interaction is clearly attractive, since if the strings overlap, the region of space where ρ is different from its value in the vacuum is reduced relative to the situation when the strings are far apart.

The situation is therefore very similar to that in superconductivity. The confining strings in the weakly coupled and strongly coupled regimes behave like Abrikosov vortices in the superconductor of the second and first kind respectively. This observation has an immediate implication for the string tension of the adjoint string. As we have discussed in the previous section the phase χ changes from 0 to 2π inside the adjoint string. The adjoint string therefore can be pictured as two fundamental strings running along each other. In the weak coupling regime the two fundamental strings repel each other. The two fundamental strings within the adjoint string therefore will not overlap, and the energy of the adjoint string is twice the energy of the fundamental one.

$$\sigma_{Adj} = 2\sigma_F \,. \tag{4.11}$$

More generally a string connecting two charges of magnitude n (in units of the fundamental charge) will split into n fundamental strings and its string tension scales as n

$$\sigma_n = n\sigma_F \,. \tag{4.12}$$

This is precisely what was found in [13] in the analysis of the doubly charged string in the Georgi–Glashow model.

In the strongly coupled region the situation is quite different. The strings attract. It is clear that the contribution of ρ to the energy will be minimized if the strings overlap completely. In that case the contribution of ρ in the fundamental and adjoint strings will be roughly the same. There will still be repulsion between the pseudoscalar cores of the two fundamental strings, so presumably the core energy will be doubled. We therefore have an estimate:

$$\sigma_n = \sigma_F + nO(\frac{M}{m}\sigma_F). \tag{4.13}$$

Again in the pure Yang–Mills limit the situation is more complicated. The scalar is lighter, therefore the interaction at large distances is attractive. However the pseudoscalar core size and its contribution to the tension is not small. In other words M/m is a number of order one. We expect therefore a dependence on n which is intermediate between eq. (4.12) and eq. (4.13)[4]

$$\sigma_n = f(n)\sigma_F, \qquad 1 < f(n) < n\,. \tag{4.15}$$

Of course this form should be valid only for strings shorter than the critical length. Adjoint strings must break if they are too long, so for very large charge separations $f(n)$ vanishes for even n and is equal to 1 for odd n. In fact it is a nontrivial question whether the critical length is large enough so that one can sensibly speak about the adjoint string at all. In the weakly coupled case we saw that in the critical length is parametrically larger than the string width. The same simple estimate in the strongly coupled region tells us that the width of the string is given by the scalar glueball mass, whereas the critical length is determined by the relation

$$L_c\sigma = 2M_G\,, \tag{4.16}$$

where M_G is the mass of the so called gluelump, or in our language the core energy of the soliton. One expects that the mass of the gluelump is of the same order as the glueball mass itself. From the lattice calculations of [16] we learn that $\sigma = .05M^2$. So it would appear that

$$L_c \sim 10M^{-1} \gg M^{-1} \tag{4.17}$$

and the string should be indeed well identifiable even though there is no apparent large parameter in the game.

5. Discussion

In this paper we have analyzed the structure of the confining string as it emerges from the low energy effective Lagrangian description of 2+1 dimensional $SU(2)$ gauge theory. The mechanism that drives the appearance of the string is common in the weak and the strong coupling regimes. Fundamental strings are the domain walls that separate the two degenerated vacua, as envisioned by 't Hooft. Adjoint strings on the other hand are somewhat different. They arise due to the fact that the phase of the disorder parameter has to have a winding number one around a point where the

[4]We would like to mention that some lattice measurements of the string tension [19] suggest the presence of the "Casimir" scaling

$$\sigma_n = \frac{n}{2}\left(\frac{n}{2}+1\right)\sigma_F. \tag{4.14}$$

From the point of view of the effective Lagrangian approach this dependence is extremely unnatural. The discrepancy may be due to the fact that the string tension measurements in 2+1 dimensions are notoriously difficult. This is due to the difficulty of separating the linear part of the potential from the Coulomb part which is logarithmic and thus very long range. It is possible that the Casimir scaling is a property of 3+1 dimensional theories but not the 2+1 dimensional ones. In fact more recent lattice simulations [20] in 2+1 dimensions cast doubt on the earlier results in this respect. It is also possible that even in 3+1 dimensions the Casimir scaling is the property of the potential in some intermediate region of distances where the energy is still dominated by the Coulomb potential (which certainly scales according to the Casimir law) rather than by the string tension.

external adjoint charge is placed. The appearance of the adjoint string is not tied so closely to the double vacuum degeneracy. Nevertheless in the weak coupling limit this connection is quite strong. This is due to the fact that the fundamental strings repel each other and therefore the adjoint string splits into two fundamental ones running parallel to each other.

We argued that the strong coupling regime representative of the pure Yang Mills theory corresponds to a different situation in which the scalar and the pseudoscalar (the radial part and the phase of the disorder parameter) have masses of the same order, but the scalar is the lighter one between the two. In this case the fundamental strings attract. The two fundamental strings "forming" the adjoint string overlap strongly in this case and practically loose their identity. The qualitative difference between the weak and strong coupling regimes is therefore similar to the difference between the superconductors of first and second kind.

Historically there have been two distinct proposals of how to understand confinement in 2+1 dimensions: 't Hooft's Z_N vortex condensation and Polyakov's screening of the monopole plasma. The former argumentation relies on the vacuum degeneracy in a crucial way. The latter one does not particularly care about it. In fact Polyakov's potential for the phase χ in compact QED is $\cos\chi$ rather than $\cos 2\chi$ [14] and therefore does not even exhibit the Z_2 symmetry. Still the string tension is nonzero. Each one of these approaches has its drawbacks. The 't Hooft mechanism came under criticism because it can not explain the adjoint string tension. On the other hand Polyakov's approach suggests that the adjoint string is absolutely stable and does not break which is obviously incorrect [13].

The effective Lagrangian approach nicely synthesizes the two. In this framework it is clear that the vacuum degeneracy is crucial for understanding the fundamental string. On the other hand the adjoint string is perfectly happy even if the vacuum state is unique, since it involves an integer winding of V. The adjoint string breaks if it is long enough, since the theory supports dynamical solitons that can screen the external adjoint charges. In this sense one could perhaps say that the mechanisms for the fundamental and adjoint string formation are distinct: 't Hooft's mechanism is relevant for the fundamental string, while Polyakov's mechanism for the adjoint string. The two are however closely related. Both are represented in a very simple and intuitive way in the effective Lagrangian through the properties of the interaction of the disorder field V.

Although in this paper we considered only the $SU(2)$ gauge theory, the discussion can be generalized to any $SU(N)$ group [11]. In this case the effective theory has a Z_N symmetry rather than Z_2. The fundamental string is still the domain wall between different vacua whereas the adjoint string involves an integer winding of the order parameter and therefore has the same vacuum state on both sides.

One can perhaps use similar ideas to explore confinement in other gauge theories not necessarily based on the $SU(N)$ group. The general structure of the "topological" mechanism of confinement as displayed in the effective Lagrangian is the following. Suppose the theory has a continuos global symmetry G broken spontaneously down to H. If $\pi_1(G/H) \neq 1$ the theory has topological solitons which carry an integer topological charge Q. If in addition G is slightly broken explicitly these solitons are linearly confined because the topological flux emanating from a soliton is squeezed into a flux tube by the explicit symmetry breaking term. This flux tube however breaks by creating a soliton - antisoliton pair from the vacuum if it is too long. This is the picture of confinement of adjoint charges. The same mechanism confines fundamental charges if the underlying nonabelian theory has dynamical fundamental matter. In a theory without fundamental matter the explicit breaking of G is not complete but a discreet subgroup G' remains intact. When G' is broken spontaneously in the vacuum the set of vacuum states is discreet. In this case the theory sustains linelike defects–domain walls between different vacua. Fundamental charges correspond to externally created states with fractional topological charge and serve as termination points for the domain walls. The fundamental strings are stable since the theory does not have dynamical fractionally charged solitons.

Finally we want to emphasize the following very important point. The dual Lagrangian was constructed starting with 't Hoofts idea of the Z_N disorder parameter which is a dual variable in the original theory. The final expression however can equally well (and in fact perhaps more correctly) be thought of as just a standard low energy effective Lagrangian of the theory. It is a local Lagrangian written in terms of the lightest excitation fields, both in the weak and in the strong coupling regimes. As such it should exhibit all the low energy phenomena, including confinement, which it does as we have seen. In this sense 't Hoofts insight about the Z_2 dual symmetry present in the theory should be viewed as a symmetry argument which constrains the form of the low energy effective Lagrangian, or in a way determines its universality class.

Recently a similar approach has been suggested in 3+1 dimensional gauge theory [21]. It was shown that a simple theory of a scalar and a symmetric tensor field under certain conditions sustains stringlike solutions which in [21] were interpreted as confining strings. This theory is naturally viewed as the low energy effective theory of 3+1 dimensional pure Yang Mills theory written in terms of the two lowest glueball fields, which in this case are a scalar and a symmetric tensor [22]. It would be extremely interesting to use the analog of 't Hooft symmetry arguments in 3+1 dimensions to constrain the form of the effective theory and to see whether the particular conditions required for existence of the string are imposed on the effective theory by these symmetry arguments.

Acknowledgments

We have benefited from interesting discussions with Luigi Del Debbio, Jeff Greensite and Mike Teper. A.K. is supported by PPARC Advanced fellowship. B.R. is grateful to the members of the Oxford Theoretical Physics Department for hospitality and to Royal Society for financial support during his visit in April–May 1998 when this work was done.

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